PHYSICS 191/193
Introductory Physics I

Equations of Kinematics and Dynamics

I. Vector Relationships ($\sin\theta = \text{opp}/\text{hyp}$; $\cos\theta = \text{adj}/\text{hyp}$; $\tan\theta = \sin\theta/\cos\theta$; $|\hat{i}| = 1$)

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k} \qquad\qquad |\vec{A}| \equiv A = \sqrt{(A_x)^2 + (A_y)^2 + (A_z)^2}$$

$$\vec{A}\cdot\vec{B} = A\ B\ \cos\varphi = A_xB_x + A_yB_y + A_zB_z$$

$$\vec{A}\times\vec{B} = \left(A\ B\ |\sin\varphi|\right)\ \hat{n} \qquad \text{(direction of } \hat{n} \text{ determined by the RH rule)}$$

$$\vec{A}\times\vec{B} = (A_yB_z - A_zB_y)\hat{i} + (A_zB_x - A_xB_z)\hat{j} + (A_xB_y - A_yB_x)\hat{k}$$

II. Equations of Kinematics (Constant Acceleration in One Dimension; $g = 9.8\ \text{m/s}^2$)

$$x = x_0 + \langle v_x\rangle t \tag{1}$$

$$v_{fx} = v_{ox} + a_xt \tag{2}$$

$$\langle v_x\rangle = (v_{fx} + v_{ox})/2 \tag{3}$$

$$x = x_0 + v_{ox}t + \frac{1}{2}a_xt^2 \tag{4}$$

$$v_{fx}^2 = v_{ox}^2 + 2a_x\Delta x \tag{5}$$

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}; \qquad \vec{v} = d\vec{r}/dt; \qquad \vec{a} = d\vec{v}/dt$$

III. Equations of Dynamics (Including Laws of Conservation; $+y$ direction is upwards.)

$$\sum\vec{F} = m\vec{a} = \left(\frac{d\vec{p}}{dt}\right) \qquad \text{where} \quad \vec{p} = m\vec{v}_{cm}$$

$$F_{spring} = -k(x - x_{eq}) \qquad f_k = \mu_kF_N \qquad f_s \le \mu_sF_N$$

$$F_{grav} = -\frac{GM_1M_2}{r^2} \qquad G = 6.67\times10^{-11}\ \text{N kg}^{-2}\ \text{m}^2$$

Work-Energy Theorem: $W_{\sum F} = \Delta K$

For constant forces: $W_{\sum F} = \sum \vec{F} \cdot \Delta \vec{r} = \sum \left(F \ \Delta r \ \cos\varphi\right)$

Energy Conservation: $\Delta K + \Delta U + \Delta U_{int} = 0$

$K = \frac{1}{2}mv^2$, $U_{grav} = mgy$, $U_{spring} = \frac{1}{2}k(x - x_{eq})^2$, and $\Delta U_{int} = f_k \Delta r > 0$.

Momentum Conservation: $\sum_{i=1}^{n} \vec{p}_{oi} = \sum_{i=1}^{n} \vec{p}_{fi}$

where n is the number of bodies in an <u>isolated</u> system.

Impulse: $\vec{J} = \Delta\vec{p} = \int \vec{F} \ dt = \left\langle \vec{F} \right\rangle \Delta t$

IV. Equations of Rotational Motion (Including Laws of Conservation)

Note that the five equations of kinematics can be used with the following substitutions:
$x \to \theta$, $v \to \omega$, and $a \to \alpha$. Also note that $\omega \equiv d\theta / dt$ and $\alpha \equiv d\omega / dt$.
Furthermore, the following "analogies" can be made:

$$m \to I, \quad F \to \tau, \quad p \to L$$

$$\therefore \quad \vec{\tau} = \vec{r} \times \vec{F} \qquad \vec{L} = m\,(\vec{r} \times \vec{v})$$

and for finite bodies: $\sum \vec{L} = I\vec{\omega}$ and $\sum \vec{\tau} = I\vec{\alpha} = \left(\dfrac{d\vec{L}}{dt}\right)$

where

$$I = \sum_{i=1}^{n} m_i \ r_i^2 = \int r^2 \ dm \qquad K_{rot} = \frac{1}{2} I \omega^2$$

For rolling motion: $v_{cm} = \omega\ r$ and $a_{cm} = \alpha\ r$. For circular motion, $\omega = 2\pi f$, $P = 1/ f$.
$\Delta s = r\Delta\theta$ (where Δs is the arclength). The centripetal acceleration experienced by a particle moving along on a circlar arc is:

$$a_{cen} = v^2 / r = \omega^2 r$$

where the acceleration vector points radially inwards, and $v = v_{\tan} = \omega$r . Also $a_{\tan} = \alpha$r .